

2006: Math I

1) $x + 5y + 4x = 180$ ①

$4x + 8y = 180$ ②

Re-arranging:

$$\begin{array}{r} x + 5x + 5y = 180 \Rightarrow 20x + 20y = 720 \\ \times 5 \quad 4x + 8y = 180 \quad \quad \quad 20x + 40y = 900 \\ \hline -20y = -180 \\ y = 9 \end{array}$$

$$\Rightarrow 5x = 135$$

$$x = 27$$

$$x = 27, y = 9$$

2)

a) sum 3: $\frac{1}{3} + \frac{1}{15} + \frac{1}{35} = \frac{35}{105} + \frac{7}{105} + \frac{3}{105} = \frac{45}{105} = \frac{9}{21} = \frac{3}{7}$

b) sum 4: $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \frac{1}{7 \times 9} = \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \frac{1}{63} = \frac{1260}{2835} = \frac{4}{9}$

c) sum 1 = $\frac{1}{3}$, sum 2 = $\frac{2}{5}$, sum 3 = $\frac{3}{7}$, sum 4 = $\frac{4}{9}$ $\frac{n}{2n+1}$ = $\frac{n^{\text{th}}}{\text{term}}$

sum 50 = $\frac{50}{2(50)+1} = \frac{50}{101}$

d) $2n+1 = 257$

$2n = 256$

$n = 128$

3) $H^2 = \frac{25}{w\sqrt{D}}$

a) $H^2 = \frac{25}{4.2\sqrt{9.5}}$

$H^2 = 1.931207$

$H = 1.3897$

or sum 7 = sum 3 + $\frac{1}{7 \times 9}$

= $\frac{3}{7} + \frac{1}{63}$

= $\frac{27}{63} + \frac{1}{63} = \frac{28}{63} = \frac{4}{9}$

$$b) 2.3^2 = \frac{25}{W\sqrt{5.1}}$$

$$5.29 = \frac{25}{W\sqrt{5.1}}$$

$$5.29\sqrt{5.1}W = 25$$

$$W = \frac{25}{5.29\sqrt{5.1}} = 2.09$$

$$W = 2.09$$

$$c) 2.9^2 = \frac{25}{3.6\sqrt{D}}$$

$$8.41 = \frac{25}{3.6\sqrt{D}}$$

$$30.276\sqrt{D} = 25$$

$$\sqrt{D} = 0.82574$$

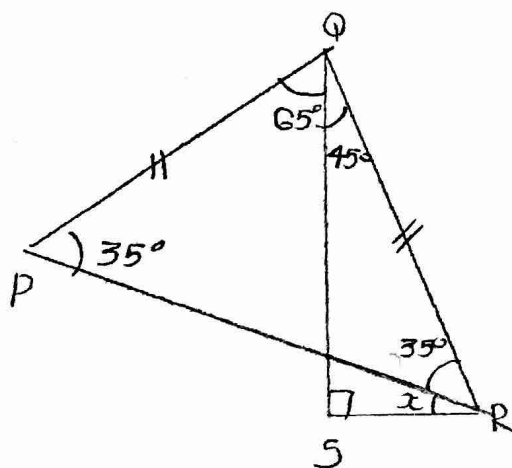
$$D = 0.6818$$

$$4) \text{ Water} = .99(650) = 643.5 \text{ grams}$$

$$\text{Squeezed out: } .95(650) = 617.5 \text{ grams}$$

$$643.5 - 617.5 = 26 \text{ grams of water}$$

5)



$$a) \angle QPP \text{ and } \angle QRP = \frac{180 - 110}{2} = 35^\circ$$

$$90 + (35 + x) + 45 = 180$$

$$\therefore x = 180 - 170 = 10^\circ$$

$$\angle QPR \text{ and } \angle QRP = \frac{180 - (a+b)}{2} = \frac{180 - a - b}{2}$$

$$90 + b + \frac{180 - a - b}{2} + x = 180$$

$$\frac{180 - a - b}{2} + b + x = 90$$

$$180 - a - b + 2b + 2x = 180$$

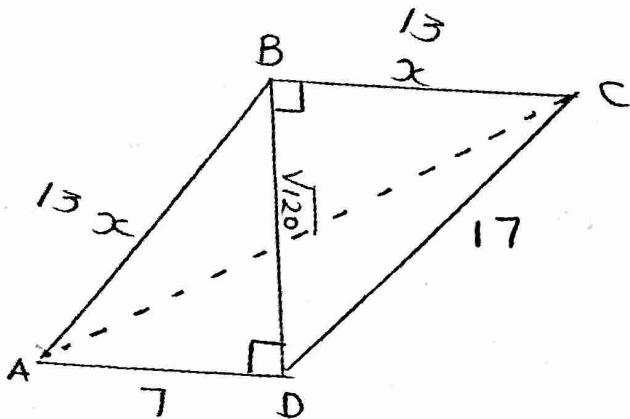
$$180 - a + b + 2x = 180$$

$$-a + b + 2x = 0$$

$$2x = a - b$$

$$x = \frac{a - b}{2}$$

6)



$$a) 7^2 + BD^2 = x^2$$

$$BD^2 = x^2 - 49$$

$$b) x^2 + BD^2 = 17^2$$

$$BD^2 = 289 - x^2$$

$$c) x^2 - 49 = 289 - x^2$$

$$2x^2 = 338$$

$$x^2 = 169$$

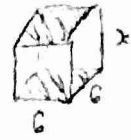
$$x = 13$$

$$d) BD = \sqrt{120}$$

$$7^2 + 17^2 = 18.38$$

i) a) When $x=0$ it means there is no side length (ie no cuboid) exists. Therefore there is no volume.

ii) When $x=6$ then volume = 0
 Not possible to have S.A. of 72 since 2 faces already have area of 72



$$2(6^2) = 72$$

b) $y = \frac{1}{2}x(36-x^2)$

When $x=4$: $y = 2(36-16) = 2(20) = 40$

c) $x=0$: $y=0$

$x=1$: $y=17.5$

$x=2$: $y=32$

$x=3$: $y=40.5$

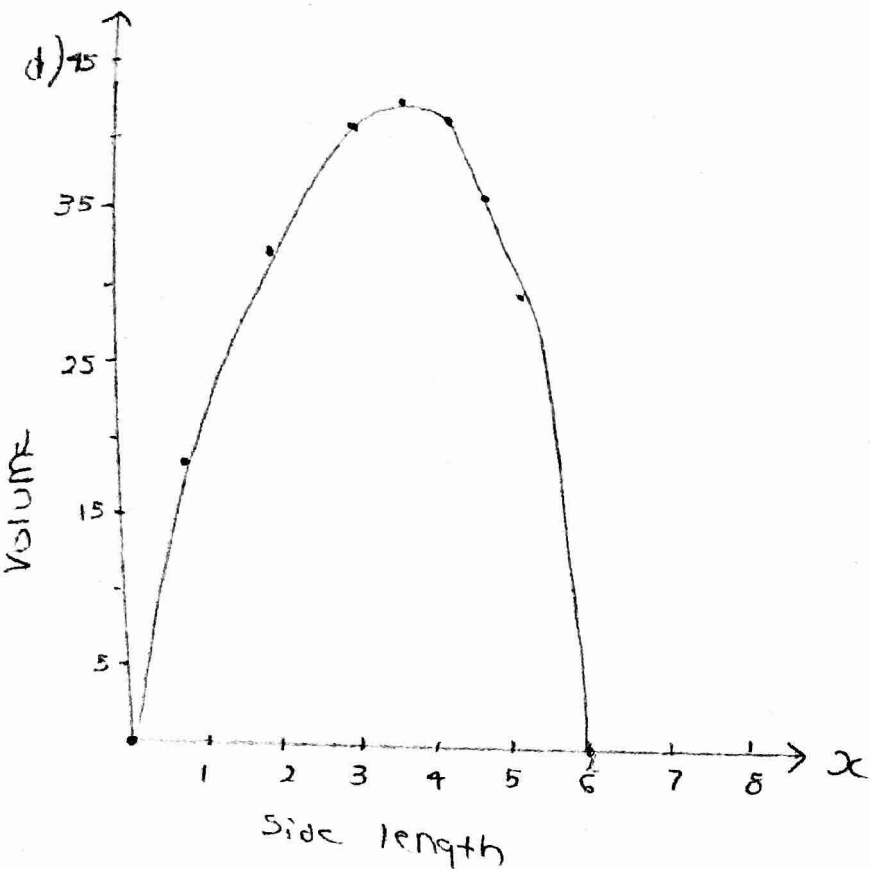
$x=3.5$: $y=41.5625$

$x=4$: $y=40$

$x=4.5$: $y=35.4375$

$x=5$: $y=27.5$

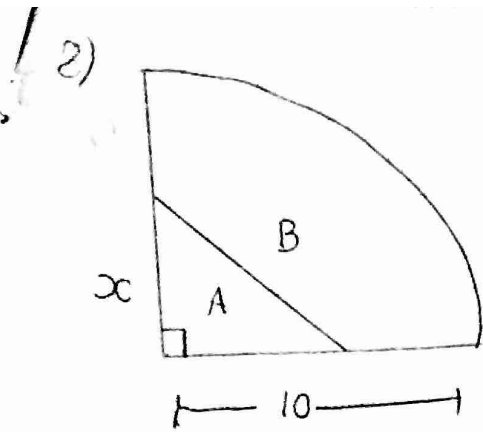
$x=6$: $y=0$



e) $x = 3.4641012$

f)





a) Area A: $\frac{1}{2}(x)(x) = \frac{1}{2}x^2$

Area B: $\frac{\pi r^2}{4} - \frac{1}{2}x^2$

$\frac{\pi r^2}{4} - \frac{1}{2}x^2 = \frac{1}{2}x^2 \Rightarrow \frac{100\pi}{4} = x^2$

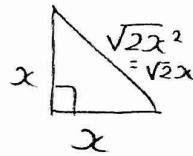
$\frac{\pi r^2}{4} = x^2$

$100\pi = 4x^2$

$x^2 = 25\pi$

$x = 5\sqrt{\pi} = 8.86$

b) Perimeter A: $x + x + \sqrt{2}x$
 $= 2x + \sqrt{2}x$



$x^2 + x^2 = y^2$
 $y = \sqrt{2x^2}$

Perimeter B: $(10-x) + (10-x) + \frac{2\pi r}{4} + \sqrt{2}x$

$= 20 - 2x + \frac{\pi r}{2} + \sqrt{2}x$

$= 20 - 2x + \frac{10\pi}{2} + \sqrt{2}x$

$= 20 - 2x + 5\pi + \sqrt{2}x$

$20 - 2x + 5\pi + \sqrt{2}x = 2x + \sqrt{2}x$

$-4x + 20 + 5\pi = 0$

$4x = 20 + 5\pi$

$x = \frac{20 + 5\pi}{4}$

$x = 8.927$

9/ a) $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 11^2$

$0, 1, 4, 9, 25, 36, 49, 64, 81, 100, 121$

• $1 = 1^2 - 0^2$

• (2)

• $3 = 2^2 - 1^2$

• $4 = 2^2 - 0^2$

• $5 = 3^2 - 2^2$

• (6)

• $7 = 4^2 - 3^2$

• $8 = 3^2 - 1^2$

• $9 = 6^2 - 5^2$ or $5^2 - 4^2$ or $3^2 - 0^2$

• (10)

• $11 = 6^2 - 5^2$

• (12) $12 = 4^2 - 2^2$

• $13 = 7^2 - 6^2$

• (14)

• $15 = 4^2 - 1^2$ or $8^2 - 7^2$

• $16 = 4^2 - 0^2$

• $17 = 9^2 - 8^2$

• (18)

• $19 = 10^2 - 9^2$

• $20 = 6^2 - 4^2$

b) $2, 6, 10, 14, 18$

$4n - 2$

c)

i) $4n - 2 = 123$

$4n = 125$

$n = \frac{125}{4} = 31.25$ NO

iv) $4n - 2 = 123456$

$4n = 123458$

$n = 30864.5$ NO

ii) $4n - 2 = 1234$

$4n = 1236$

$n = \frac{1236}{4} = 309$ YES

iii) $4n - 2 = 12345$

$4n = 12347$

$n = \frac{12347}{4} = 3086.75$ NO